Nonlocal two–dimensional Yang–Mills- and generalized Yang–Mills-theories

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Abstract

A generalization of the two–dimensional Yang–Mills and generalized Yang–Mills theory is introduced in which the building B–F theory is nonlocal in the auxiliary field. The classical and quantum properties of this nonlocal generalization are investigated and it is shown that for large gauge groups, there exist a simple correspondence between the properties a nonlocal theory and its corresponding local theory.

1 Introduction

Pure two–dimensional Yang–Mills theories (YM_2) have certain properties, such as invariance under area–preserving diffeomorphisms and lack of any propagating degrees of freedom. There are, however, ways to generalize these theories without losing these properties. One way is the so–called generalized Yang–Mills theories (gYM_2's). In a YM_2, one starts from a B–F theory in which a Lagrangian of the form $i\mathrm{tr}(BF) + \mathrm{tr}(B^2)$ is used. Here F is the field–strength corresponding to the gauge–field, and B is an auxiliary field in the adjoint representation of the gauge group. Carrying a path integral over this field, leaves an effective Lagrangian for the gauge field of the form $\mathrm{tr}(F^2)$ [1]. In a gYM_2, on the other hand, one uses an arbitrary class function of the auxiliary field B, instead of $\mathrm{tr}(B^2)$ [2]. In [3] the partition function and the expectation values of the Wilson loops for gYM_2's were calculated. It is worthy of mention that for gYM_2's, one can not eliminate the auxiliary field and obtain a Lagrangian for the gauge field. One can, however, use standard path–integration and calculate the observables of the theory. This was done in [4].

To study the behaviour of these theories for large groups is also of interest. This was studied in [5] and [6] for ordinary YM_2 theories and then in [7] for YM_2 and in [8] and [9] for gYM_2 theories. It was shown that YM_2's and some classes of gYM_2's have a third–order phase transition in a certain critical area.

There is another way to generalize YM_2, and that is to use a non–local action for the auxiliary field. An example of such theories is one in which the B^2 action is replaced by an action

$$S_B := w\left[\int \mathrm{d}\mu\ \mathrm{tr}(B^2)\right], \tag{1}$$

or by

$$S_B := w\left[\int \mathrm{d}\mu\ \Lambda(B)\right], \tag{2}$$

where Λ is a class function. We call these gerneralizations nonlocal YM_2 ($nlYM_2$) and nonlocal gYM_2 ($nlgYM_2$), respectively. We want to investigate the classical, and quantum behaviour of these theories, and also their properties for large groups.

The scheme of the present paper is the following. In section 2, we introduce the $nlYM_2$ and show that the classical solutions of YM_2 are also classical solutions of $nlYM_2$.

In section 3, the wave functions and, as a special case, the partition function of $nlYM_2$ on general surfaces are calculated.

In section 4, the properties of $nlYM_2$ large groups are investigated, and it is shown that these properties can be related to those of YM_2 by simple redefinitions.

Finally, in section 5 we introduce $nlgYM_2$ theories, and study their properties, including the wave functions and large–group behaviour. It is seen that

the large–group properties of nlgYM$_2$ are easily obtained from those of ordinary gYM$_2$, in a manner similar to the case of nlYM$_2$.

2 nlYM$_2$: definition and the classical properties

We define the nlYM$_2$ with the following action.

$$e^S := \int \mathrm{D}B\ \exp\left\{\int \mathrm{d}\mu\ i\mathrm{tr}(BF) + w\left[\int \mathrm{d}\mu\ \mathrm{tr}(B^2)\right]\right\}. \tag{3}$$

Here $\mathrm{d}\mu$ is the invariant measure of the surface:

$$\mathrm{d}\mu := \frac{1}{2}\epsilon_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu, \tag{4}$$

B is a pseudo–scalar field in the adjoint representation of the group, and F is the field strength corresponding to the gauge field. The classical equation for the gauge field is

$$\frac{\delta S}{\delta A} = 0, \tag{5}$$

or equivalently,

$$\frac{\delta}{\delta A}e^S = 0. \tag{6}$$

To obtain this, we note that

$$\frac{\delta}{\delta F}e^S = i\int \mathrm{D}B\ B\exp\left\{\int \mathrm{d}\mu\ i\mathrm{tr}(BF) + w\left[\int \mathrm{d}\mu\ \mathrm{tr}(B^2)\right]\right\}. \tag{7}$$

The integral in the right–hand side is not easy to be carried out. But, using the fact that the only tensor appearing in the right–hand side is the *metric*

$$\mathcal{G}_{ab}(x,y) := \omega_{ab}\delta(x,y), \tag{8}$$

where ω is the metric defined on the algebra, it is seen that there exists a function $\mathcal{A}$ such that

$$\frac{\delta}{\delta F}e^S = F\mathcal{A}\left[\int \mathrm{d}\mu\ \mathrm{tr}(F^2)\right], \tag{9}$$

or in a more explicit form,

$$\frac{\delta}{\delta F^a(x)}e^S = F_a(x)\mathcal{A}\left[\int \mathrm{d}\mu\ \mathrm{tr}(F^2)\right]. \tag{10}$$

From this, one can obtain the left–hand side of (6) as

$$\frac{\delta}{\delta A^a(x)}e^S = \int \mathrm{d}\mu' \frac{\delta}{\delta F^b(x')}e^S \frac{\delta F^b(x')}{\delta A^a(x)}$$

$$= \quad \mathcal{A}\left[\int \mathrm{d}\mu \; \mathrm{tr}(F^2)\right] \int \mathrm{d}\mu' \; F_b(x') \frac{\delta F^b(x')}{\delta A^a(x)}. \tag{11}$$

Comparing this to the variation of the YM action, it is seen that

$$\frac{\delta}{\delta A^a(x)} e^S = -2\mathcal{A}\left[\int \mathrm{d}\mu \; \mathrm{tr}(F^2)\right] \frac{\delta}{\delta A^a(x)} S^{\mathrm{YM}}. \tag{12}$$

This shows that classical solutions of the YM theory are also classical solutions of the nlYM theory.

3 The partition function and the wave functions of nlYM's

Along the lines of [1, 4], we begin by calculating the wave–function of a disk. We have

$$\begin{aligned} \psi_D(U) &= \int \mathrm{D}F \; e^S \delta\left(\mathrm{Pexp} \oint_{\partial D} A, U\right) \\ &= \int \mathrm{D}B \; \mathrm{D}F \exp\left\{\int \mathrm{d}\mu \; i\mathrm{tr}(BF) + w\left[\int \mathrm{d}\mu \; \mathrm{tr}(B^2)\right]\right\} \\ &\quad \times \delta\left(\mathrm{Pexp} \oint_{\partial D} A, U\right). \end{aligned} \tag{13}$$

Here U is the class of the Wilson loop corresponding to the boundary. The delta function is also a class delta function, that is, its support is where its two arguments are in the same congugacy class. This delta function can be expanded in terms of the characters of irreducible unitary representations of the group:

$$\delta\left(\mathrm{Pexp} \oint_{\partial D} A, U\right) = \sum_R \chi_R(U^{-1}) \chi_R\left(\mathrm{Pexp} \oint_{\partial D} A\right). \tag{14}$$

One can now introduce Fermionic variables η and $\bar{\eta}$ in the representation R to write the Wilson loop as [4]

$$\chi_R\left(\mathrm{Pexp} \oint_{\partial D} A\right) = \int \mathrm{D}\eta \; \mathrm{D}\bar{\eta} \left[\int_0^1 \mathrm{d}t \; \bar{\eta}(t)\dot{\eta}(t) + \oint_{\partial D} \bar{\eta} A \eta\right] \eta^\alpha(0)\bar{\eta}_\alpha(1). \tag{15}$$

Inserting (15) in (14) and then (13), using the Schwinger–Fock gauge, and integrating over, F, B, and the fermionic variables, respectively, one arrives at

$$\psi_D(U) = \sum_R \chi_R(U^{-1}) d_R \exp\{w[-AC_2(R)]\}, \tag{16}$$

where d_R is the dimension of the representation R and $C_2(R)$ is the second Casimir of the representation R. The details of calculation are the same as those done in [4]. Note, however, that one cannot simply glue the disk wave–functions to obtain, for example, the wave function corresponding to a larger disk or that of a sphere. The reason is that the action of the original B–F theory is not extensive, that is

$$S_{A_1+A_2}(B,F) \neq S_{A_1}(B,F) + S_{A_2}(B,F). \tag{17}$$

So, if the disk D is divided into two smaller disks D_1 and D_2,

$$\psi_D(U) \neq \int \mathrm{d}U_1\ \psi_{D_1}(UU_1)\psi_{D_2}(UU_1^{-1}). \tag{18}$$

Two obtain the wave function for an arbitrary surface, however, one can begin with a disk of the same area and impose boundary conditions on certain parts of the boundary of the disk. These conditions are those corresponding to the identifications needed for constructing the desired surface from a disk. This is done in exactly in the same manner as the case of YM_2 or gYM_2, as the only things to be calculated are integrations over group of characters of the same representation [10]. This is easily done and one arrives at

$$\begin{aligned}\psi_{\Sigma_{g,q}}(U_1,\cdots,U_n) &= \sum_R h_R^q d_R^{2-2g-q-n} \\ &\quad \times\chi_R(U_1^{-1})\cdots\chi_R(U_n^{-1})\exp\{w[-C_2(R)A(\Sigma_{g,q})]\}\end{aligned} \tag{19}$$

Here $\Sigma_{g,q}$ is a surface containing g handles and q projective planes. It has also n boundaries. This is the most general surface with finite area. h_R is defined through

$$h_R := \int \mathrm{d}U \chi_R(U^2), \tag{20}$$

and it is zero unless the representation R is self–conjugate. In this case, this representation has an invariant bilinear form. Then, $h_R = 1$ if this form is symmetric and $h_R = -1$ if it is antisymmetric [11].

As a special result, the partition function of the theory on a sphere is obtained if we put U_i's equal to unity and q and g equal to zero. We arrive at

$$Z = \sum_R d_R^2 \exp\{w[-AC_2(R)]\}. \tag{21}$$

4 Large–N limit of nlYM$_2$

Starting from (21), consider the case the gauge group is $U(N)$. The representations of this group are labeled by N integers n_i satisfying

$$n_i \geq n_j, \qquad i < j. \tag{22}$$

The dimension of this representation is

$$d_R = \prod_{1\le i<j\le N} \left(1 + \frac{n_i - n_j}{j-i}\right), \tag{23}$$

and the l-th Casimir is

$$C_l(R) = \sum_{i=1}^{N} [(n_i + N - i)^l - (N-i)^l]. \tag{24}$$

For $l = 2$, one can redefine the function w and introduce another function:

$$w(-AC_2) =: -N^2 W(A\tilde{C}_2), \tag{25}$$

where

$$\tilde{C}_l(R) := \frac{1}{N^{l+1}} \sum_{i=1}^{N} (n_i + N - i)^l. \tag{26}$$

Then, following [5], we use the definitions

$$x := i/N, \tag{27}$$

and

$$\phi := \frac{i - n_i - N}{N}, \tag{28}$$

to write the partition function as

$$Z = \int \mathrm{D}\phi\ \exp[S(\phi)], \tag{29}$$

where

$$S(\phi) := -N^2 \left\{ W\left[A \int_0^1 \mathrm{d}x\ \phi^2(x)\right] + \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\ \log|\phi(x) - \phi(y)| \right\}. \tag{30}$$

In the large–N limit, only the representation (i.e. the configuration of ϕ) contributes to the partition function that maximizes S. To find it, one puts the variation of S with respect to ϕ equal to zero:

$$-A\, W'\left[A \int \mathrm{d}y\ \phi^2(y)\right] \times 2\phi(x) + 2\mathrm{P} \int \frac{\mathrm{d}y}{\phi(x) - \phi(y)} = 0. \tag{31}$$

Defining

$$\tilde{A} := A\, W'\left[A \int \mathrm{d}y\ \phi^2(y)\right], \tag{32}$$

the equation for ϕ is written as

$$\frac{\tilde{A}}{2}[2\phi(x)] = \mathrm{P}\int \frac{\mathrm{d}y}{\phi(x)-\phi(y)}. \tag{33}$$

This equation is the same as that obtained in [5] and [6], and can be solved in the same manner. First, one defines a density function for ϕ as

$$\rho(z) := \frac{\mathrm{d}x}{\mathrm{d}\phi(x)}\Big|_{\phi(x)=z}. \tag{34}$$

Then, (33) becomes

$$\frac{\tilde{A}}{2}(2z) = \mathrm{P}\int_{-a}^{a} \frac{\mathrm{d}w\ \rho(w)}{z-w}. \tag{35}$$

Next, a function H is defined on the complex plane through

$$H(z) = \int_{-a}^{a} \frac{\mathrm{d}w\ \rho(w)}{z-w}. \tag{36}$$

Here, z is a complex variable. This function is analytic on the complex plane, except for $z \in [-a, a]$, where H has a branch cut. Also, from the definition of ρ, it is seen that

$$\int_{-a}^{a} \mathrm{d}w\ \rho(w) = 1, \tag{37}$$

which shows

$$H(z) \sim \frac{1}{z}, \qquad z \to \infty. \tag{38}$$

The function H is then calculated to be [6]

$$H(z) = \frac{1}{2\pi i}\sqrt{z^2-a^2}\oint_c \frac{(\tilde{A}/2)(2\lambda)\mathrm{d}\lambda}{(z-\lambda)\sqrt{\lambda^2-a^2}}, \tag{39}$$

where the integration contour encircles the cut $[-a, a]$ but the point z is outside it. The integration is readily done and one arrives at

$$H(z) = \tilde{A}(z - \sqrt{z^2-a^2}). \tag{40}$$

To obtain a, one can use (38), which yields

$$a = \sqrt{\frac{2}{\tilde{A}}}. \tag{41}$$

From (36), it is seen that

$$-\pi\rho(z) = \mathrm{Im}H(z+i\epsilon), \qquad \text{for } z \text{ real}, \tag{42}$$

which gives

$$\rho(z) = \frac{\tilde{A}}{\pi}\sqrt{\frac{2}{\tilde{A}} - z^2}. \tag{43}$$

This is, of course, in complete accordance with [5, 6]. But one must now obtain the density in terms of A not $\tilde{A}$. To do so, let us return to the definition (32). First, we need the integral

$$\int \mathrm{d}x\ \phi^2(x) = \int \mathrm{d}z\ \rho(z)z^2, \tag{44}$$

which is the coefficient of $1/z^3$ in the large–z expansion of H. This is calculated to be

$$\begin{aligned} \int \mathrm{d}z\ \rho(z)z^2 &= \frac{\tilde{A}a^4}{8} \\ &= \frac{1}{2\tilde{A}}. \end{aligned} \tag{45}$$

From this, one obtains an equation for $\tilde{A}$ as

$$\tilde{A} = A\, W'\left(\frac{A}{2\tilde{A}}\right), \tag{46}$$

or

$$\frac{Aa^2}{4}W'\left(\frac{Aa^2}{4}\right) = \frac{1}{2}. \tag{47}$$

Defining a free–energy function as

$$f := -\frac{1}{N^2}S\Big|_{\phi_{\mathrm{cla.}}}, \tag{48}$$

It is seen that

$$\begin{aligned} f'(A) &= \int \mathrm{d}x\ \phi^2(x)W'\left[A\int \mathrm{d}y\ \phi^2(y)\right] \\ &= \frac{\tilde{A}}{A}\int \mathrm{d}x\ \phi^2(x). \end{aligned} \tag{49}$$

or

$$f'(A) = \frac{1}{2A}. \tag{50}$$

The function W is disappeared from f', as it can be seen by the rescaling $\tilde{\phi} := \sqrt{A}\phi$.

This completes our discussion of the weak–region nlYM. As A increases, a situation is encountered where ρ exceeds 1. This density function is, however,

not acceptable, as it violates the condition (22). The value of A at which this occurs is obtained from

$$\rho(0) = 1, \tag{51}$$

which gives

$$\tilde{A}_c = \frac{\pi^2}{2}. \tag{52}$$

For $\tilde{A} > \tilde{A}_c$, one must take an ansatz for ρ as

$$\rho_s(z) = \begin{cases} \tilde{\rho}_s(z), & z \in L \\ 1, & z \in [-b, b] \end{cases}, \tag{53}$$

where

$$L := [-a, -b] \cup [a, b] \tag{54}$$

Using methods exactly the same as those used in [6, 7, 8], one must solve

$$\frac{\tilde{A}}{2}(2z) = \mathrm{P} \int_{-a}^{a} \frac{\mathrm{d}w\ \rho_s(w)}{z - w}, \qquad z \in L. \tag{55}$$

To do so, one defines a function H_s as

$$H_s(z) = \int_{-a}^{a} \frac{\mathrm{d}w\ \rho_s(w)}{z - w}, \tag{56}$$

which is found to be

$$\begin{aligned} H_s(z) &= \log\frac{z+b}{z-b} + \frac{\sqrt{(z^2-a^2)(z^2-b^2)}}{2\pi i} \\ &\quad \times \oint_{c_L} \mathrm{d}\lambda \frac{(\tilde{A}/2)(2\lambda) - \log[(\lambda+b)/(\lambda-b)]}{(z-\lambda)\sqrt{(\lambda^2-a^2)(\lambda^2-b^2)}} \\ &= \frac{\tilde{A}}{2}(2z) - \sqrt{(z^2-a^2)(z^2-b^2)} \\ &\quad \times \int_{-b}^{b} \frac{\mathrm{d}\lambda}{(z-\lambda)\sqrt{(a^2-\lambda^2)(\lambda^2-b^2)}}. \end{aligned} \tag{57}$$

Here c_L is a contour encircling L, leaving $[-b, b]$ and the point z out. Note that everything is exactly the same as the case of ordinary YM theory, except that A is replaced by $\tilde{A}$. Using the fact that H_s should behave as $1/z$ for large z, one obtains two equations

$$\tilde{A} = \int_{-b}^{b} \frac{\mathrm{d}\lambda}{\sqrt{(a^2-\lambda^2)(\lambda^2-b^2)}} \tag{58}$$

and

$$1 = \int_{0}^{b} \mathrm{d}\lambda \frac{a^2+b^2-\lambda^2}{\sqrt{(a^2-\lambda^2)(\lambda^2-b^2)}}. \tag{59}$$

For $\tilde{A}$ near $\tilde{A}_c$, these equations are solved as

$$\frac{1}{a} = \frac{\pi}{2}\left(1 + \frac{\pi^2 b^2}{16} + \frac{\pi^4 b^4}{128}\right) \tag{60}$$

and

$$\tilde{A} = \frac{\pi^2}{2}\left(1 + \frac{\pi^2 b^2}{8} + \frac{7\pi^4 b^4}{256}\right). \tag{61}$$

Now, using (49), and the fact that the integral $\int \mathrm{d}x\ \phi^2(x)$ is the coefficient of $1/z^3$ in the large–z expansion of H_s, one arrives at

$$f_s'(A) = \frac{\tilde{A}}{A}\left[\frac{1}{2\tilde{A}} + \frac{(\tilde{A}-\tilde{A}_c)^2}{\tilde{A}_c^3}\right] + O[(\tilde{A}-\tilde{A}_c)^3], \tag{62}$$

or

$$\begin{aligned} f_s' - f_w' &= \frac{(\tilde{A}-\tilde{A}_c)^2}{A_c\tilde{A}_c^2} \\ &= \left(\frac{\mathrm{d}\tilde{A}}{\mathrm{d}A}\right)^2_{c,s} \frac{(A-A_c)^2}{A_c\tilde{A}_c^2}. \end{aligned} \tag{63}$$

This shows that the third–order phase transition is there, unless the derivative of $\tilde{A}$ with respect to A is zero at the critical point in the strong region.

5 nlgYM$_2$: wave functions, partition function, and the large–N limit

A nonlocal generalized Yang–Mills theory is defined by the action

$$e^S := \int \mathrm{D}B\ \exp\left\{\int \mathrm{d}\mu\ i\mathrm{tr}(BF) + w\left[\int \mathrm{d}\mu\ \Lambda(B)\right]\right\}. \tag{64}$$

Following [4], and using the same technique of section 3, one can find the wave function on a disk:

$$\psi_D(U) = \sum_R \chi_R(U^{-1}) d_R \exp\{w[AC_\Lambda(R)]\}, \tag{65}$$

where

$$C_\Lambda(R)1_R := \Lambda(-iT_R), \tag{66}$$

and $\Lambda(-iT_R)$ means that one has put $-iT^a$ in the representation R instead of B^a in the function Λ. Using the same technique of section 3, one can construct the wave function on an arbitrary surface as

$$\psi_{\Sigma_{g,q}}(U_1, \cdots, U_n) \quad = \quad \sum_R h_R^q d_R^{2-2g-q-n}$$

$$\times\chi_R(U_1^{-1})\cdots\chi_R(U_n^{-1})\exp\{w[C_\Lambda(R)A(\Sigma_{g,q})]\}, \quad (67)$$

and the partition function on the sphere as

$$Z = \sum_R d_R^2 \exp\{w[AC_\Lambda(R)]\}. \quad (68)$$

The large–N limit of this theory (for the gauge group $U(N)$), is defined by suitable rescalings. Taking C_Λ a linear function of the rescaled Casimirs (26), one can define a function W as

$$-N^2 W\left[A\sum_l \alpha_l \tilde{C}_l(R)\right] := w[AC_\Lambda(R)]. \quad (69)$$

In the large–N limit, the partition function becomes like (29), but with

$$S(\phi) := -N^2\left\{W\left[A\int_0^1 \mathrm{d}x\; G(\phi)\right] + \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\;\; \log|\phi(x)-\phi(y)|\right\}, \quad (70)$$

where

$$G(\phi) := \sum_l (-1)^l a_l \phi^l. \quad (71)$$

Then, following the procedure of section 4, that representation contributes to the partition function that maximizes S. The equation for this representation is

$$-A\,W'\left[A\int \mathrm{d}y\; G(\phi)\right] G'[\phi(x)] + 2\mathrm{P}\int \frac{\mathrm{d}y}{\phi(x)-\phi(y)} = 0. \quad (72)$$

Defining

$$\begin{aligned} \tilde{A} &:= A\,W'\left\{A\int \mathrm{d}y\; G[\phi(y)]\right\} \\ &= A\,W'\left[A\int \mathrm{d}z\; \rho(z)\; G(z)\right], \end{aligned} \quad (73)$$

the equation for ϕ is written as

$$\frac{\tilde{A}}{2}G'[\phi(x)] = \mathrm{P}\int \frac{\mathrm{d}y}{\phi(x)-\phi(y)}, \quad (74)$$

or, in terms of the density ρ,

$$\frac{\tilde{A}}{2}G'(z) = \mathrm{P}\int \frac{\mathrm{d}w\;\rho(w)}{z-w}. \quad (75)$$

Note that equation (75) does not contain W. That is, the solution for ρ is the same as that obtained for gYM$_2$. The only difference is that one should put $\tilde{A}$

instead of A. This is true for the weak region, as well as for the strong region. One can also obtain the derivative of the free energy with respect to A, in the same way it was obtained in section 4. We have

$$\begin{aligned} f'(A) &= \int \mathrm{d}x\, G[\phi(x)]W'\left\{A\int \mathrm{d}y\, G[\phi(y)]\right\} \\ &= \frac{\tilde{A}}{A}\int \mathrm{d}z\, \rho(z)\, G(z). \end{aligned} \tag{76}$$

The set of equations to be solved are (75) and (37), to obtain ρ in terms of $\tilde{A}$; then (73) to obtain $\tilde{A}$ in terms of A. Also note that $f'(A)$ is essentially the same as $f'(A)$ for the corresponding *local* gYM_2. The differences are the existence of $\tilde{A}$ instead of A, and an overall factor $\tilde{A}/A$. The similarity between the nonlocal theory and the local theory holds also in the strong region. Since there, one still has equation (75) for the set where $\rho < 1$. That is, here too the equations for ρ are the same as those of the corresponding local theory.

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